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PAL tolerances analysis: a proposed strategy

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Summary

Formulae are derived by means of which the quantities actually to be transmitted, using the PAL system, can be determined when the 1960 C.I.E.-U.C.S. chromaticity co-ordinates and the luminance of the colour to be transmitted are known. Errors in toleranced quantities will cause corresponding errors in the received values of the quantities actually transmitted, and the above-mentioned formulae are 'reversible' so that the received displayed luminance and chromaticity co-ordinates can be uniquely deduced.

When these errors are known, the corresponding impairment can be estimated (in terms of just-noticeable-difference units); two alternative methods of estimation are considered, and applied to the case of simultaneous errors of chroma-gain (g) and of output gamma ($\Delta\gamma$). Explicit approximate formulae (containing linear and quadratic terms in g and $\Delta\gamma$) were obtained from which the resulting impairment could be easily derived. The maximum impairment which must be taken into account can thus be deduced, knowing the tolerances applicable to g and $\Delta\gamma$. Results obtained for this particular case are discussed. The extension of this analysis to the case when several toleranced quantities are in error simultaneously is outlined. Errors contributed by various toleranced quantities leading to large-area errors in displayed luminance and colour co-ordinates are briefly considered. The maximum relevant impairment can be approximately calculated as if each toleranced quantity had an error numerically equal to 90% of its tolerance, and detailed knowledge is not required of the statistical distribution of toleranced quantities, or of the way in which such distributions should be combined.

List of Principal Symbols

g	Chroma-gain error ($100g$%)
$R_oG_oB_o$	Colour separation components of transmitted waveform. (Components in the down-gamma state will be referred to as R_o^{1/γ_1}, G^{1/γ_2}, etc.)
R, G, B	Colour separation components of received waveform
u_o, v_o	1960 C.I.E.-U.C.S. chromaticity co-ordinates of colour to be transmitted
u_D, v_D	Corresponding co-ordinates of Illuminant D_{65} (u_D = 0·1978; v_D = 0·3122)
u, v	Corresponding co-ordinates of received displayed colour
$U_oV_oW_o$	Tristimulus values associated with the 1960 C.I.E.-U.C.S. chromaticity co-ordinates for the colour to be transmitted. (V_o is the relative luminance, so that $V_o = 1$ when $R_o = G_o = B_o = 1$)
U, V, W	Corresponding values associated with the received displayed colour (the receiver is assumed to be specified in terms of Equations (10) throughout; V is the relative luminance, so that $V = 1$ if $R = G = B = 1$)
$U_o^*V_o^*W_o^*$	Co-ordinates for the colour to be transmitted in Wyszecki colour space [defined by Equations (4) to (6)]
$U^*V^*W^*$	Wyszecki colour-space co-ordinates for the received displayed colour
γ_1	Value of gamma (taken as 2·8 in Section 5) assumed to be used in forming transmitted signal
γ_2	Cathode ray tube gamma. (The correct value of γ_2 is also taken as 2·8 in Section 5)
$\Delta C_u = 260(u-u_o)$	Error in u-co-ordinate of received colour signal, expressed in j.n.d. (just noticeable difference) units
$\Delta C_v = 260(v-v_o)$	Corresponding error in v-co-ordinate of received colour signal, in j.n.d. units

$$\Delta E = \left\{(\Delta U^*)^2 + (\Delta V^*)^2 + (\Delta W^*)^2\right\}^{1/2}$$

$$\Delta E_k = \left\{(\Delta L)^2 + (\Delta C_u)^2 + (\Delta C_v)^2\right\}^{1/2}$$

(Both ΔE and ΔE_k are estimates of the overall subjective error, in j.n.d. units; the suffix k refers to the (ΔL, ΔC_u, ΔC_v) colour space used at Kingswood Warren)

$\Delta L = 116 \log_{10}(V/V_o)$	Luminance error in j.n.d. units
$\Delta U^*, \Delta V^*, \Delta W^*$	$U^*-U_o^*$, $V^*-V_o^*$ and $W^*-W_o^*$ respectively
$\Delta\gamma$	$\gamma_2-\gamma_1$ (error in output gamma)
ξ_o, η_o, Y'_o ξ, η, Y'	The quantities actually transmitted (with suffix 0) and received (with no suffix). (The symbols U and V normally used for describing the colour difference components of the chrominance signal have not been used, to avoid confusion with the luminance and related quantities specified above in the list)

1. Introduction

In the PAL colour television transmission system, the present practice is to ensure that certain quantities are each controlled within specified limits. But at present these limits or tolerances are decided somewhat arbitrarily. If a tolerance is unnecessarily tight, needless trouble and expense may be incurred in the operation of colour television transmission networks, while if a tolerance is not tight enough excessive impairment of the picture may result. Hitherto, however, the effect of toleranced quantities on the picture output has not been understood. Several investigators have determined the size of the 'just noticeable difference' (j.n.d.) over the 1960 C.I.E.-U.C.S. diagram: some results have been summarised by Sproson[1] who has shown that there are considerable differences between various determinations of the magnitude of this quantity. Here the MacAdam value is taken in which 1 j.n.d. is equivalent to a vector length of 0·00384 on the diagram. In describing errors of luminance in j.n.d. values, a 2% change of luminance is taken as 1 j.n.d. The overall j.n.d. error is obtained by taking a 'root sum of squares' combination of the chromaticity and luminance errors.[2]

An alternative method of expressing colorimetric errors involves the use of 'Wyszecki colour space'.[3] This is discussed in Section 2.

The main result of the present investigation is a method of estimating quantitatively (in terms of j.n.d. units) the worst overall effect of a number of toleranced quantities simultaneously in error which is likely to occur with non-negligible probability. This method of error estimation is based upon the concept of the way in which the errors arise illustrated in Fig. 1(*a*).

We suppose that the 1960 C.I.E.-U.C.S. chromaticity co-ordinates of the colour to be transmitted are (u_o, v_o), and that the associated luminance is V_o as indicated at the top left hand corner of Fig. 1(*a*). Because various toleranced quantities are in error, the received chromaticity co-ordinates are (u, v) and the received luminance is V as indicated on the right of Fig. 1(*a*).†

Fig. 1(*a*) indicates the way in which the differences between u_o, v_o and V_o on the one hand and u, v and V respectively on the other arise and Figs. 1(*b*) and 1(*c*) indicate two alternative ways of estimating the subjective effects of these differences.

† Fig. 1(*a*) assumes that the scene is illuminated by light of the same colour temperature as that of the white point to which the monitor is adjusted (i.e. D_{65}). This is not in general true of colour transmission, but the simplification helps in presenting the case and does not upset the general validity of the argument.

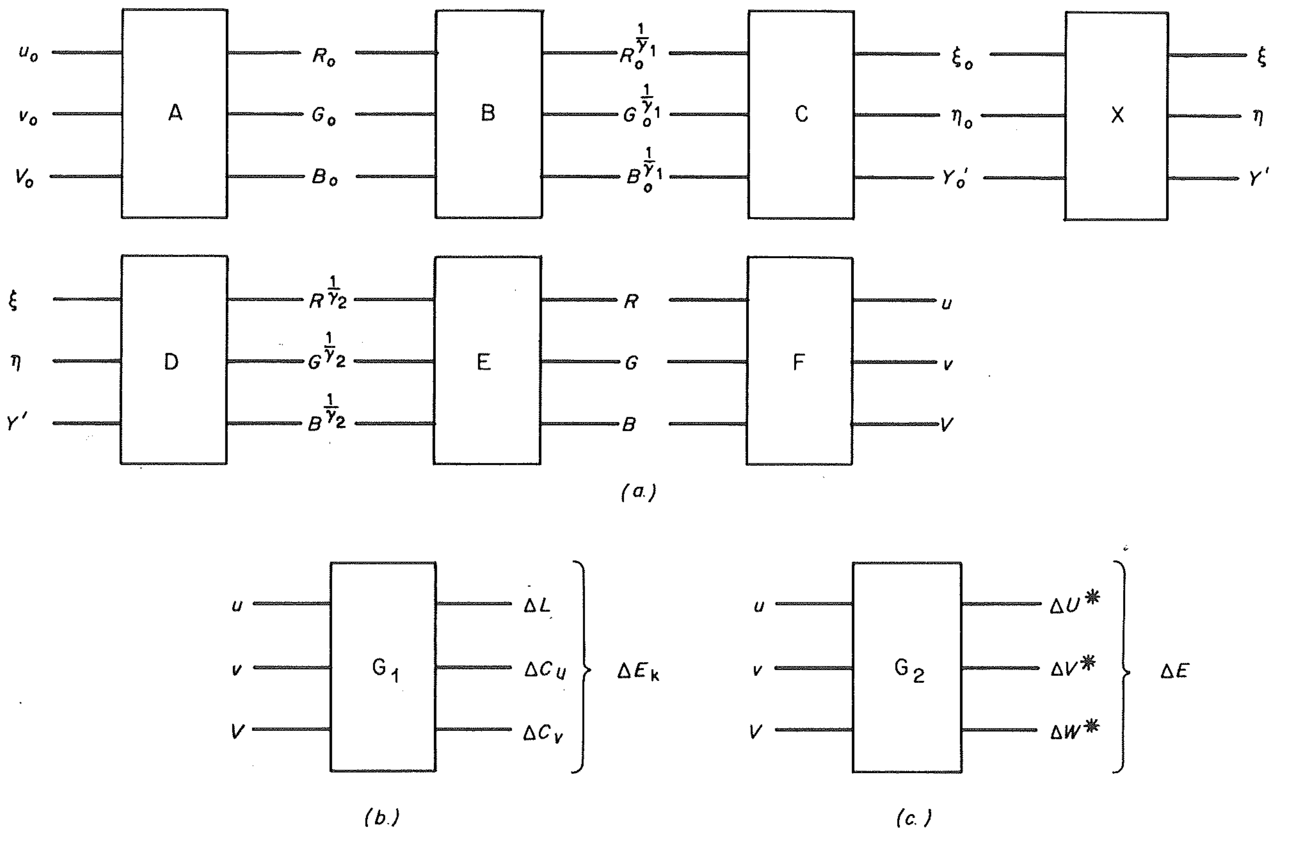

Fig. 1 - Generation and subjective effect of output chromaticity and luminance errors
(*a*) Generation of output chromaticity and luminance errors
(*b*) Subjective effect of chromaticity and luminance errors: for a simple colour space
(*c*) Subjective effect of chromaticity and luminance errors: for Wyszecki colour space

Now the quantities actually transmitted are not u_o, v_o and V_o but ξ_o, η_o and Y'_o defined below, and indicated in Fig. 1(a) between the 'black boxes' C and X, so it is first necessary to appreciate the mechanism whereby ξ_o, η_o and Y'_o are obtained. The black box A represents the fact that given u_o, v_o and V_o, the corresponding colour separation components R_o, G_o and B_o are in principle uniquely determined. Black boxes A and B constitute the camera and associated equipment; they are together here regarded simply as devices by means of which the red component of the original colour gives rise to an output R_o^{1/γ_1} where $1/\gamma_1$ is the camera gamma and similarly with the green and blue components. C is a real and existent 'black box' which gives outputs Y'_o, ξ_o and η_o (the quantities actually transmitted) in response to inputs R_o^{1/γ_1}, G_o^{1/γ_1}, and B_o^{1/γ_1}. The next box X is the principal source of errors of the kind we shall consider in this study. Any particular toleranced quantity T in error is likely to contribute errors corresponding to the quantities $(\xi - \xi_o)$, $(\eta - \eta_o)$ and $(Y' - Y'_o)$; such contributions are represented in Fig. 1(a) by the black box X. The signal received is thus specified by quantities ξ, η and Y' of the same kind as ξ_o, η_o and Y'_o but having different values. Some errors in toleranced quantities may contribute additional and separate errors to γ_1 and γ_2. Any change of the error in T can usually but not necessarily be regarded as changing the resulting errors in $(\xi - \xi_o)$ etc. proportionally. Again, if several toleranced quantities are in error simultaneously, it will be assumed (at any rate as a first approximation) that the total error in each of $(\xi - \xi_o)$ etc. is correctly obtained by adding the errors contributed by each individual toleranced quantity. But to obtain any useful information from this assumption, our first objective must be to express results in terms of the displayed chromaticity co-ordinates u, v and the received luminance V.

Now there is a notable symmetry about Fig. 1(a): if all the toleranced quantities had their correct values, the box X would do nothing, ξ would equal ξ_o, η would equal η_o, Y' would equal Y'_o and γ_2 would equal γ_1. Hence the black box D in Fig. 1 would merely 'undo' what the black box C had done, the black box E† would then undo what B had done, and the black box F† would undo what A had done; we should finish with the correct colour co-ordinates (u_o, v_o) and the correct relative luminance V_o. We now exercise a mathematician's privilege of 'working backwards', and saying that, in the general case, any signal which is associated with the red-emitting part of the display tube and associated equipment, and denoted as R must have arrived at the input of the display tube as R^{1/γ_2} and similarly for the other colour separation components. Hence if we continue to 'work backwards', the black box D is doing the same as black box C in the forward direction. The one apparently significant difference is that approaching black box C it is R_o^{1/γ_1}, G_o^{1/γ_1} and B_o^{1/γ_1} that are known and ξ_o, η_o and Y'_o that are required, whereas approaching black box D it is ξ, η and Y' that are given and R^{1/γ_2}, G^{1/γ_2} and B^{1/γ_2} that are required. But it will be shown that a knowledge of ξ, η and Y' uniquely determines R^{1/γ_2}, G^{1/γ_2} and B^{1/γ_2} and vice versa. It remains to derive an estimate of the subjective effects of receiving u, v and V when u_o, v_o and V_o were transmitted, and this can be done either in terms of ΔE_k for a simple form of colour space or in terms of ΔE for the more sophisticated 'Wyszecki' colour space. For the simple colour space, the black box G_1 is only a theoretical entity whereby ΔL, ΔC_u and ΔC_v are determined when u, v and V are known; ΔE_k is then at once deduced. For 'Wyszecki' colour space, on the other hand, black box G_1 must be replaced by G_2 whereby ΔU^*, ΔV^* and ΔW^* are determined when u, v and V are known, and ΔE is then at once deduced. Quantitative estimates of subjective picture quality by means of the quantities ΔE_k or ΔE are considered further in Section 2.

† E and F can be regarded as constituting the display tube.

In Section 3, formulae are derived for determining ΔE_k and its components in terms of the colour separation components R, G and B corresponding to the displayed picture and the given colour separation components R_o, G_o and B_o of the transmitted picture (or other given quantities associated with the transmitted picture from which R_o, G_o and B_o can be calculated). In Section 4, formulae are derived and discussed for expressing R, G and B in terms of the quantities actually transmitted and vice versa. These formulae are perfectly general and not related to the particular toleranced quantity under consideration. The above ideas are applied to the case when chroma-gain and output gamma are simultaneously in error in Section 5. Results for this case are discussed in Section 6. Section 7 deals with the general case in outline, in terms of the changes in ξ, η, Y', γ_1 and γ_2, while Section 8 considers briefly the errors in these quantities due to various toleranced quantities.

2. Quantitative estimate of subjective picture quality

We shall consider in what follows two methods of estimating the impairment of picture quality by means of objective measurements or calculations. The more straightforward of these methods involves the errors in the luminance and in the 1960 C.I.E.-U.C.S. chromaticity co-ordinates of the displayed colour. Let the displayed (tristimulus) value of the relative luminance be V, normalised so that white as displayed by the phosphors under consideration corresponds to $V = 1$. Let the transmitted relative luminance be V_o, normalised so that $V_o = 1$ at white when $R_o = G_o = B_o = 1$. Then the component ΔL of impairment due to luminance error is taken as given by the formula

$$\Delta L = \left\{\log_{10} V/V_o\right\}/\log_{10} 1{\cdot}02 \approx 116 \log_{10}(V/V_o) \quad (1)$$

Again, if (u, v) are the 1960 C.I.E.-U.C.S. chromaticity co-ordinates of the displayed colour, while (u_o, v_o) are the corresponding co-ordinates of the colour to be transmitted, then the components ΔC_u, ΔC_v of impairment due to colour-co-ordinate errors are taken as given by

$$\left.\begin{aligned}\Delta C_u &= (u - u_o)/0{\cdot}00384 \approx 260(u - u_o)\\ \Delta C_v &= (v - v_o)/0{\cdot}00384 \approx 260(v - v_o)\end{aligned}\right\} \quad (2)$$

and the overall estimate of impairment is ΔE_k where†

$$\Delta E_k = \left\{\Delta L^2 + \Delta C_u^{\,2} + \Delta C_v^{\,2}\right\}^{1/2} \quad (3)$$

† The suffix k is used to distinguish ΔE_k from ΔE defined by Equation (8) below; the k is for Kingswood Warren, where the simple $(\Delta L, \Delta C_u, \Delta C_v)$ colour space has been used extensively to date.

This is illustrated in Fig. 2, where P_o is the point (in 'simple colour space') corresponding to the transmitted colour, so that $\Delta L = \Delta C_u = \Delta C_v = 0$ there; the axes are in three mutually perpendicular directions representing ΔL, ΔC_u and ΔC_v respectively, and the vector P_oP (marked with a double arrow) has magnitude ΔE_k. The corresponding diagram can be drawn for ΔE given by Equation (8) below in Wyszecki space.

It may sometimes be useful to regard the magnitude of the vector ΔE_k as a signed quantity, having the same sign as that of its numerically largest component (ΔL, ΔC_u or ΔC_v as the case may be). For our present purpose, we are not greatly concerned with the sign of the magnitude of ΔE_k, but rather with the greatest absolute value of ΔE_k which is likely to be encountered. The quantities ΔL, ΔC_u, ΔC_v and ΔE_k are all deemed to be measured in 'just noticeable difference' units (j.n.d.).

The second alternative is to carry out the corresponding procedure in 'Wyszecki[3] colour space (1964-C.I.E.)'. The co-ordinates of the point (U^*, V^*, W^*) in this space, corresponding to a point (associated with the displayed colour) with displayed luminance V and colour co-ordinates (u, v), are given by the equations

$$U^* = 13W^* (u - u_D) \tag{4}$$

$$V^* = 13W^* (v - v_D) \tag{5}$$

$$W^* = 25\,(100V)^{1/3} - 17 \quad (1 \geqslant V \geqslant 0{\cdot}01) \tag{6}$$

where (u_D, v_D) are the colour co-ordinates of Illuminant D_{65}. Similar equations with U^* replaced by U_o^*, u replaced by u_o, etc. give the co-ordinates U_o^*, V_o^*, W_o^* of the Wyszecki colour space point corresponding to the colour to be transmitted. We next define

$$\left.\begin{aligned} \Delta U^* &= U^* - U_o^* \\ \Delta V^* &= V^* - V_o^* \\ \Delta W^* &= W^* - W_o^* \end{aligned}\right\} \tag{7}$$

and then we define ΔE by

$$\Delta E = \left\{(\Delta U^*)^2 + (\Delta V^*)^2 + (\Delta W^*)^2\right\}^{1/2} \tag{8}$$

instead of by (3).

It is uncertain whether ΔE_k or ΔE is the more useful quantity, but the results discussed in Section 6 give some indication that ΔE is preferable. At present we shall confine detailed theoretical analysis to Equation (3), knowing that very similar formulae are available if (8) is ultimately preferred, and that for a large-scale analysis involving computer programming, the machine times for calculations in terms of either (3) or (8) would not differ significantly.

In Section 3, we therefore consider how to derive u, v, V and hence ΔL, ΔC_u, ΔC_v and ΔE_k when R, G, B are assumed known; in Section 4 the relation between R, G, B and the quantities actually transmitted is discussed:

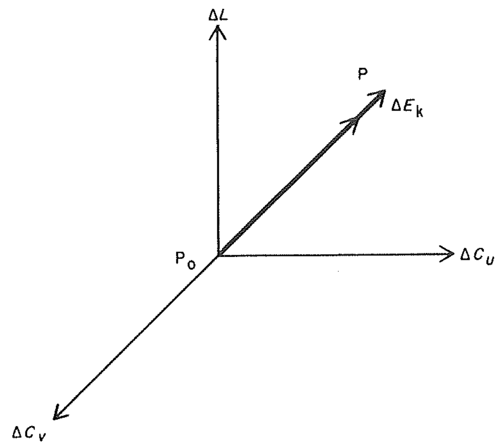

Fig. 2 - The significance of Equation (3)

this relation permits the derivation (in Section 5) of R, G and B when chroma-gain and output gamma are simultaneously in error.

3. Formulae relating output picture quality to original and displayed colour separation components

In this Section, we assume that we know the colour separation components R, G and B of the displayed picture. In practice, it may be necessary (as in the cases discussed in Sections 5 and 7) to determine those quantities first from the information available for the particular case under discussion. Knowing R, G and B (and all relevant quantities connected with the transmitted waveform) we here derive formulae for obtaining ultimately the relative luminance (V), and the chromaticity co-ordinates of the displayed colour, from which we can derive ΔC_u, ΔC_v and ΔL by means of Equations (1) and (2). We suppose that the luminances corresponding to the original and displayed colours are respectively V_o and V, while the 1960 C.I.E.-U.C.S. chromaticity co-ordinates of the original and displayed colours are respectively (u_o, v_o) and (u, v). One way of defining chromaticity co-ordinates in terms of tristimulus values is as follows:

$$\frac{U_o}{u_o} = \frac{V_o}{v_o} = \frac{W_o}{1 - u_o - v_o} \; : \; \frac{U}{u} = \frac{V}{v} = \frac{W}{1 - u - v} \tag{9}$$

Further, it can be shown that the linear relationship between U, V, W on the one hand, and R, G and B on the other, is given by the single matrix equation (for CCIR System I parameters)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 0{\cdot}2863 & 0{\cdot}2289 & 0{\cdot}1185 \\ 0{\cdot}2215 & 0{\cdot}7074 & 0{\cdot}0711 \\ 0{\cdot}1270 & 0{\cdot}9552 & 0{\cdot}4874 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix} \tag{10}$$

and that the corresponding equation also holds when U, V, W are respectively replaced by U_o, V_o and W_o while R, G, B are respectively replaced by the corresponding colour

separation components R_o, G_o, B_o of the original picture. Equations (9) and (10) are equivalent to†

$$\left.\begin{aligned} R &= (V/v)\,(\;\;4{\cdot}847674u + 0{\cdot}973653v - 0{\cdot}950777) \\ G &= (V/v)\,(-1{\cdot}475330u + 1{\cdot}669280v + 0{\cdot}082926) \\ B &= (V/v)\,(-0{\cdot}423511u - 5{\cdot}576837v + 2{\cdot}136926) \end{aligned}\right\} \quad (11)$$

or to

$$V = 0{\cdot}2215R + 0{\cdot}7074G + 0{\cdot}0711B \qquad (12)$$

$$u = \frac{0{\cdot}2863R + 0{\cdot}2289G + 0{\cdot}1185B}{0{\cdot}6348R + 1{\cdot}8915G + 0{\cdot}6770B} \qquad (13)$$

$$v = \frac{0{\cdot}2215R + 0{\cdot}7074G + 0{\cdot}0711B}{0{\cdot}6348R + 1{\cdot}8915G + 0{\cdot}6770B} \qquad (14)$$

Equations (11) also permit us to determine the colour separation components R_o, G_o, B_o required to transmit the colour having chromaticity co-ordinates (u_o, v_o) with luminance V_o if V_o is substituted for V, u_o for u and v_o for v on the right hand side. Equations (12), (13) and (14) permit us to carry out the reverse process for the displayed colour – determining its relative luminance V and its chromaticity co-ordinates u, v, given its colour separation components R, G and B.

Equation (12) tells us directly that the displayed luminance is V whereas we have assumed that the original luminance was V_o. Equations (13) and (14) give the chromaticity co-ordinates of the displayed colour, whereas u_o and v_o were the corresponding co-ordinates for the original colour.

Thus assuming that we are given (or have previously found) the displayed colour separation components R, G and B and that we know all the relevant quantities (having a suffix zero) associated with the transmitted waveform, we can find the displayed luminance V from (12) and the displayed chromaticity co-ordinates u, v from (13) and (14) and this part of the analysis does not depend upon the particular toleranced quantity under discussion. The toleranced quantity only affects the way in which R, G and B are derived from the information available.

4. Relations between R_o, G_o, B_o and the quantities actually transmitted

In practice, the quantities actually transmitted are not R_o, G_o and B_o but Y'_o, ξ_o and η_o defined by

$$Y'_o = lR_o^{1/\gamma_1} + mG_o^{1/\gamma_1} + nB_o^{1/\gamma_1} \qquad (15)$$

$$\xi_o = (B_o^{1/\gamma_1} - Y'_o)/2{\cdot}03 \qquad (16)$$

$$\eta_o = (R_o^{1/\gamma_1} - Y'_o)/1{\cdot}14 \qquad (17)$$

† Note that in Equations (11), the coefficients have been worked out to six decimal places to reduce rounding-off errors as much as possible. The data cannot be regarded as reliable to more than the third or fourth significant figure.

where $l + m + n = 1$. The factors 2·03 and 1·14 in Equations (16) and (17) are in accordance with reference 4, Section 5.3; the values of l, m and n in Equation (18) are given in Section 5.2 and the value 2·8 for γ_1 is in accordance with Section 2.4.

We shall assume in all numerical work which follows that γ_1 is 2·8† and

$$l = 0{\cdot}299 \qquad m = 0{\cdot}587 \qquad n = 0{\cdot}114 \qquad (18)$$

Equations (15), (16) and (17) are linear relations between Y'_o, ξ_o and η_o on one hand and R_o^{1/γ_1}, G_o^{1/γ_1} and B_o^{1/γ_1} on the other, so that if we are given R_o^{1/γ_1}, G_o^{1/γ_1} and B_o^{1/γ_1}, we can find Y'_o, ξ_o and η_o, and vice versa. More precisely, we assume that we know the colour separation components R_o, G_o and B_o associated with the original colour. (If it is the luminance V_o and chromaticity co-ordinates (u_o, v_o) of the original colour that are known, Equations (11) give the corresponding values of Y'_o, ξ_o and η_o but the received values Y', ξ and η of these quantities are in general different because of the toleranced error-causing quantities under discussion.)

5. The case when only chroma-gain and output gamma are in error

Since there is no phase distortion

$$\frac{\eta}{\xi} = \frac{\eta_o}{\xi_o} = \tan\phi_o, \text{ say} \qquad (19)$$

If the chroma-gain error is g (or $100g$%) then we know that

$$\left(\frac{\xi^2 + \eta^2}{\xi_o^{\,2} + \eta_o^{\,2}}\right)^{1/2} = 1 + g \text{ (except at black level)} \qquad (20)$$

From Equation (19) it therefore follows that

$$\frac{\xi}{\xi_o} = \frac{\eta}{\eta_o} = \left(\frac{\xi^2 + \eta^2}{\xi_o^{\,2} + \eta_o^{\,2}}\right)^{1/2} = 1 + g \qquad (21)$$

Chroma-gain error does not involve any luminance-signal distortion; from this fact and Equations (19), (20) and (21) it can be shown that

$$Y'_o = Y' = lR^{1/\gamma_2} + mG^{1/\gamma_2} + nB^{1/\gamma_2} \qquad (22)$$

$$R^{1/\gamma_2} = (1+g)R_o^{1/\gamma_1} - gY'_o \qquad (23)$$

$$G^{1/\gamma_2} = (1+g)G_o^{1/\gamma_1} - gY'_o \qquad (24)$$

$$B^{1/\gamma_2} = (1+g)B_o^{1/\gamma_1} - gY'_o \qquad (25)$$

We now suppose that g and $\gamma_2 - \gamma_1 = \Delta\gamma$ are small quantities of the first order of magnitude. Given R_o, G_o and B_o, and hence Y'_o from (15), we can deduce R, G and B from

† The artificial restriction that γ_1 = 2·8 and the correct γ_2 is also 2·8 will be removed, and the general case discussed in Section 7.

Equations (23), (24) and (25). At present we shall, as already noted, assume that $\gamma_1 = 2{\cdot}8$ in all numerical work and that the correct value of γ_2 is also 2·8. Then we can obtain an explicit formula for R in terms of known quantities correct to the second order in g and $\Delta\gamma$, namely

$$R - R_o = \delta R = R_o\gamma_1 Xg + (R_o \log_e R_o/\gamma_1)\Delta\gamma + \tfrac{1}{2}\gamma_1(\gamma_1 - 1)R_o X^2 g^2 + $$
$$+ R_o X(1 + \log_e R_o) g\Delta\gamma + \left\{R_o(\log_e R_o)^2/(2\gamma_1{}^2)\right\}\Delta\gamma^2 \qquad (26)$$

where $$X = 1 - R_o{}^{-1/\gamma_1} Y'_o \qquad (27)$$

There are similar formulae for $\delta G = G - G_o$ and for $\delta B = B - B_o$. The G-formula is obtained by substituting G_o for R_o in (26) and (27) and the B-formula is obtained by substituting B_o for R_o in (26) and (27).

It is quite a straightforward operation, capable of computer programming, to substitute from Equation (26) and the corresponding equations for δG and δB into Equations (12), (13) and (14) for any particular transmitted colour, and thus obtain explicit formulae for ΔL, ΔC_u and ΔC_v (and also ΔU^*, ΔV^* and ΔW^* if necessary) correct to the second order in g and $\Delta\gamma$ for these quantities. This has been done for four of the 26 colours used by BBC Research Department, namely Colours 9, 12, 15 and 24 with the following results:[†]

Although officially it is the value of ΔE_k given by Equation (3) which is supposed to be our estimate of subjective reaction, Equations (28) to (31) suggest that one of the components of ΔE_k will often vary much more than the remainder and thus largely control the variations of ΔE_k. In the case of Colour 9, for example, the worst value of ΔE_k will occur when $\Delta\gamma$ has its extreme value and when g has an extreme value of the same sign; ΔL is the predominant component. It is only when g and $\Delta\gamma$ have such

[†] These preliminary, exploratory results were obtained by means of a desk calculating machine. Details of the 26 colours are given, for example, in BBC Research Department Report No. 1967/59.

Colour 9

$$\left.\begin{aligned}
&R_o = 0{\cdot}2715;\ G_o = 0{\cdot}2730;\ B_o = 0{\cdot}6576;\ V_o = 0{\cdot}3000;\ u_o = 0{\cdot}1924;\ v_o = 0{\cdot}2646\\
&\Delta L = -0{\cdot}014g - 20{\cdot}968\Delta\gamma + 1{\cdot}324g^2 + 1{\cdot}623g\Delta\gamma + 0{\cdot}328(\Delta\gamma)^2\\
&\Delta C_u = -1{\cdot}453g - 0{\cdot}577\Delta\gamma + 0{\cdot}035g^2 - 0{\cdot}600g\Delta\gamma - 0{\cdot}016(\Delta\gamma)^2\\
&\Delta C_v = -13{\cdot}18g - 5{\cdot}130\Delta\gamma + 0{\cdot}245g^2 - 5{\cdot}570g\Delta\gamma - 0{\cdot}174(\Delta\gamma)^2
\end{aligned}\right\}(28)$$

Colour 12

$$\left.\begin{aligned}
&R_o = 0{\cdot}6549;\ G_o = 0{\cdot}6303;\ B_o = 0{\cdot}0972;\ V_o = 0{\cdot}5979;\ u_o = 0{\cdot}2051;\ v_o = 0{\cdot}3572\\
&\Delta L = 6{\cdot}239g - 8{\cdot}547\Delta\gamma + 1{\cdot}055g^2 + 3{\cdot}213g\Delta\gamma + 0{\cdot}131(\Delta\gamma)^2\\
&\Delta C_u = 1{\cdot}351g + 0{\cdot}423\Delta\gamma - 0{\cdot}570g^2 + 0{\cdot}041g\Delta\gamma - 0{\cdot}057(\Delta\gamma)^2\\
&\Delta C_v = 6{\cdot}492g + 1{\cdot}718\Delta\gamma - 5{\cdot}028g^2 - 1{\cdot}727g\Delta\gamma - 0{\cdot}536(\Delta\gamma)^2
\end{aligned}\right\}(29)$$

Colour 15

$$\left.\begin{aligned}
&R_o = 0{\cdot}4930;\ G_o = 0{\cdot}0742;\ B_o = 0{\cdot}2135;\ V_o = 0{\cdot}1769;\ u_o = 0{\cdot}3068;\ v_o = 0{\cdot}2959\\
&\Delta L = 14{\cdot}412g - 24{\cdot}186\Delta\gamma + 10{\cdot}357g^2 + 17{\cdot}803g\Delta\gamma + 2{\cdot}328(\Delta\gamma)^2\\
&\Delta C_u = 26{\cdot}486g + 10{\cdot}162\Delta\gamma - 6{\cdot}305g^2 + 5{\cdot}786g\Delta\gamma - 0{\cdot}621(\Delta\gamma)^2\\
&\Delta C_v = -0{\cdot}831g + 0{\cdot}366\Delta\gamma + 3{\cdot}294g^2 + 3{\cdot}297g\Delta\gamma + 0{\cdot}640(\Delta\gamma)^2
\end{aligned}\right\}(30)$$

Colour 24

$$\left.\begin{aligned}
&R_o = 0{\cdot}4649;\ G_o = 0{\cdot}2477;\ B_o = 0{\cdot}1968;\ V_o = 0{\cdot}2922;\ u_o = 0{\cdot}2376;\ v_o = 0{\cdot}3258\\
&\Delta L = 0{\cdot}695g - 21{\cdot}368\Delta\gamma + 1{\cdot}470g^2 + 1{\cdot}945g\Delta\gamma + 0{\cdot}315(\Delta\gamma)^2\\
&\Delta C_u = 11{\cdot}611g + 4{\cdot}31\Delta\gamma + 0{\cdot}988g^2 + 5{\cdot}161g\Delta\gamma + 0{\cdot}188(\Delta\gamma)^2\\
&\Delta C_v = 3{\cdot}376g + 1{\cdot}148\Delta\gamma - 0{\cdot}273g^2 + 0{\cdot}902g\Delta\gamma - 0{\cdot}050(\Delta\gamma)^2
\end{aligned}\right\}(31)$$

extreme values that there will be any point in calculating ΔE_k at all: when a calculation of ΔE_k is required, the correct procedure is to substitute numbers for g and $\Delta\gamma$ into (28) or analogous equations, and then deduce ΔE_k numerically. The linear terms of ΔL, ΔC_u and ΔC_v give the same absolute values of these quantities if g is replaced by $-g$ and $\Delta\gamma$ is simultaneously replaced by $-\Delta\gamma$; it is the quadratic terms which decide which of these cases is the worst.

We seek to determine the worst value of ΔE_k which occurs with sufficient probability for corrective action to be necessary, and we have very little information about the statistical distribution of quantities like g and $\Delta\gamma$ within their tolerances. It is therefore suggested that the worst value of ΔE_k which need be considered is that which occurs when g and $\Delta\gamma$ are numerically equal to 0·9 times their tolerance. For if the statistical distributions of g and $\Delta\gamma$ are assumed rectangular, *both* quantities will only reach simultaneously values numerically greater than 90% of tolerance for 1% of the time, and the occurrence of extreme values is much more probable with a rectangular distribution than with the best fitting equivalent Gaussian distribution.

For all of Colours 9, 12, 15 and 24, the principal terms in ΔL etc. appear to be the linear ones, although the second order terms are not negligible. If therefore a value of ΔE_k is obtained which is k times what it should be, a first approximation to the corrective action required is to reduce the tolerances of the quantities contributing most to ΔE_k in the ratio k : 1.

6. Graphical representation

It is helpful to plot the variation of ΔE_k for Colours 9, 12, 15 and 24 (Equations (28) to (31) used in conjunction with Equation (3)) for the two variables g (chroma gain error) and $\Delta\gamma$ (variation of display gamma). Results are shown as contour maps of ΔE_k in Figs. 3 – 6. (Note that ΔE_k is always zero when g and $\Delta\gamma$ are both zero.) If we examine Fig. 3 we observe that the contours have maximum diameter at an inclination of about -10° to the chroma gain error axis. The implication is that (on the scale shown in Fig. 3) chroma gain error has a smaller effect on the overall colour error than change of display gamma: further there is some tendency for errors to compensate, e.g. a gain in chroma can be partially compensated by a decrease of display gamma. The contours of ΔE_k shown in Fig. 5 on the other hand are approximately circular in shape and this implies that no partial compensation of errors is taking place and a change of 0·1 in chroma gain has much the same effect as a change of 0·1 in display gamma. Colour No. 12, Fig. 4, appears anomalous in that the direction for minimisation of the error (i.e. partial compensation) is $+45^\circ$ with reference to the g axis, i.e. simultaneous increases of chroma gain and display gamma partially cancel one another. This anomaly appears to occur because Equations (1) to (3) tend to overemphasise luminance errors relative to chromaticity errors. Although this overemphasis applies to all of Figs. 3 – 6, its marked effect in Fig. 4 is probably associated with the high value of V_o for Colour 12.

The results described above follow from the use of the colour space defined by ΔE_k in Equations (1), (2) and (3). The alternative colour space defined by Wyszecki (1964 C.I.E.) and expressed in Equations (4) to (8) gives rise to a different set of contours of ΔE and these are shown in Figs. 7 – 10. One obvious difference is the relative weightings of errors produced by changes in g or $\Delta\gamma$: the major axes of the ellipses are now about -75° with reference to the $+g$ axis and this value holds approximately for all the four colours investigated. Partial compensation of colour errors (ΔE) is given for all colours by slight increase of g with a considerable reduction in display gamma.

In view of the difference in contours predicted by the two colour spaces used, it would be very desirable to determine experimentally which of the two is more nearly representative of practical observation of colour pictures under typical viewing conditions. The mathematical treatment given in earlier sections is applicable to either colour space and for computer working there is no appreciable difference in the time taken to compute either ΔE_k or ΔE. From the point of view of practical significance and usefulness, however, it is highly desirable to compute numerical results which closely correspond to practical experience. The anomalous Fig. 4 gives some indication that ΔE may be a better indication of subjective reaction than ΔE_k.

7. The general case in outline

We have assumed in the foregoing that relationships of the form

$$\left.\begin{aligned} \Delta\xi &= \xi - \xi_o = \sum_T a_T e_T \\ \Delta\eta &= \eta - \eta_o = \sum_T b_T e_T \\ \Delta Y' &= Y' - Y'_o = \sum_T c_T e_T \end{aligned}\right\} \qquad (32)$$

can be determined, where e_T is the error in a typical toleranced quantity T, and a_T, b_T and c_T are determinate constants. We have also assumed that there may be errors in γ_1 and γ_2. These relationships are discussed for various toleranced quantities giving rise to large-area luminance and chromaticity errors in Section 8; our present concern is to derive equations analogous to (26) and (27) from which equations analogous to (28), (29), (30) and (31) can be deduced when $\Delta\xi$, $\Delta\eta$ and $\Delta Y'$ have arbitrary values but are assumed to be small quantities of the first order of magnitude. We retain quantities of the second order in $\Delta\xi$, $\Delta\eta$ and $\Delta Y'$ but discard all higher orders. The equations will also involve γ_1 and γ_2 which may be unequal.

We have already noted that Equations (15) to (17) are true as they stand, and also, by 'looking backwards' at the system from the display tube, they are true if all the suffixes zero are removed and γ_1 is replaced by γ_2. From

the six† equations, thus obtained, it can be shown that

$$R^{1/\gamma_2} = R_o^{\,1/\gamma_1} + \Delta Y' + 1{\cdot}14\Delta\eta \tag{33}$$

$$G^{1/\gamma_2} = G_o^{\,1/\gamma_1} + \Delta Y' - 0{\cdot}39424\Delta\xi - 0{\cdot}58068\Delta\eta \tag{34}$$

$$B^{1/\gamma_2} = B_o^{\,1/\gamma_1} + \Delta Y' + 2{\cdot}03\Delta\xi \tag{35}$$

We can reasonably subtract $R_o^{\,1/\gamma_2}$ from both sides of (33) and treat $(R^{1/\gamma_2} - R_o^{\,1/\gamma_2})$ and $(R_o^{\,1/\gamma_1} - R_o^{\,1/\gamma_2})$ as small quantities of the first order of magnitude and similarly for Equations (34) and (35). These equations thus play the same part that Equations (26) and (27) did in the particular case discussed in Section 5, and are applicable for any combination of toleranced quantities in error, including those which affect Y', γ_1 and γ_2. As we shall see in Section 8, the majority of errors in toleranced quantities appear to affect significantly only ξ and η.

† Equations (15) to (17) and a similar set with suffixes zero removed.

As in Section 5, we are mainly interested in extreme values of ΔE_k or ΔE which are likely to occur too often to be ignored or 'put up with'. It was suggested in Section 5 that a good rough and ready rule is to consider the case when each toleranced quantity is in error by an amount numerically equal to 90% of the maximum permitted. Acceptance of this suggestion means that it is only necessary to make numerical substitutions for a few

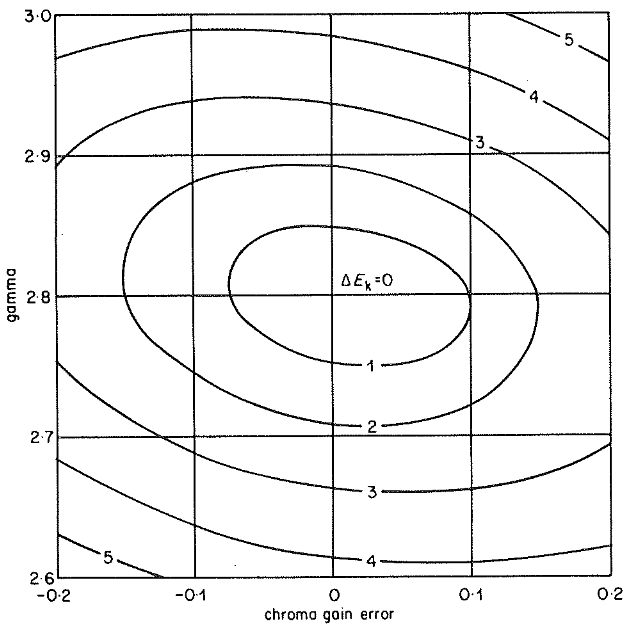

Fig. 3 - Contours of ΔE_k for Colour No. 9 in j.n.d. units (for varying chroma-gain error and display gamma)

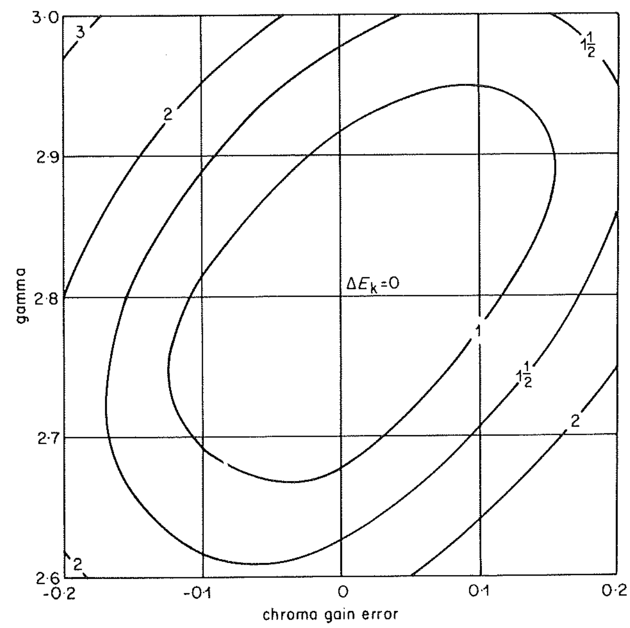

Fig. 4 - Contours of ΔE_k for Colour No. 12 in j.n.d. units (for varying chroma-gain error and display gamma)

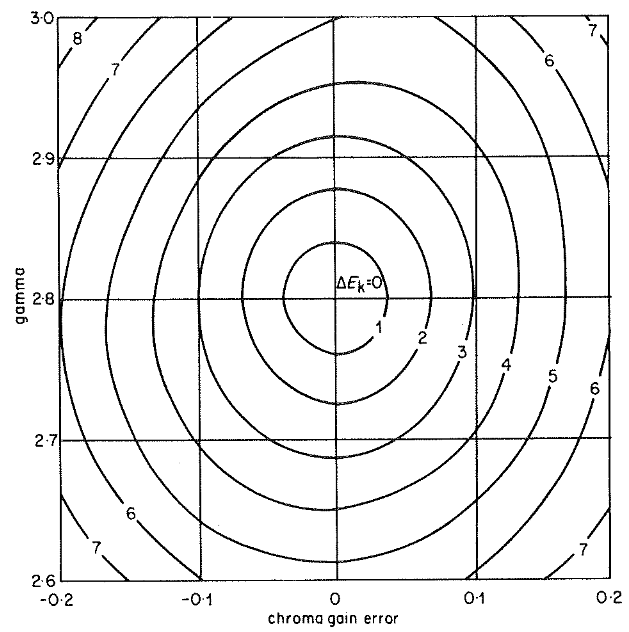

Fig. 5 - Contours of ΔE_k for Colour No. 15 in j.n.d. units (for varying chroma-gain error and display gamma)

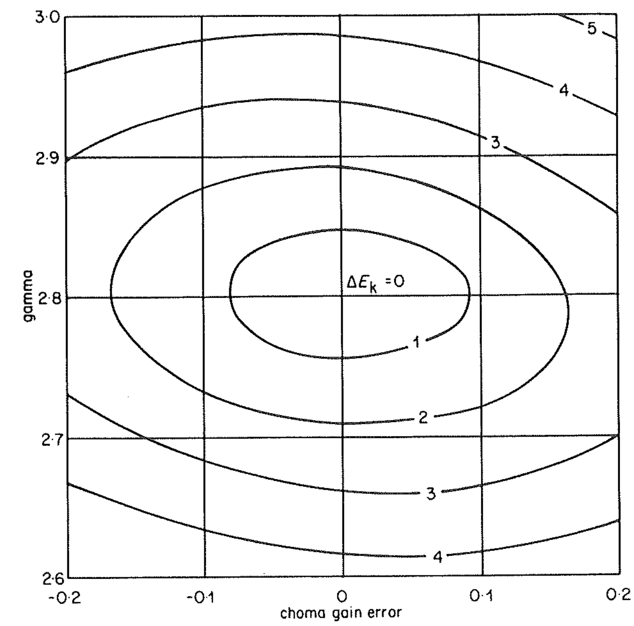

Fig. 6 - Contours of ΔE_k for Colour No. 24 in j.n.d. units (for varying chroma-gain error and display gamma)

particular cases from (33), (34) and (35) into (12), (13) and (14) instead of deriving general equations like (28) to (31) although these equations are useful to help our understanding of the general way (discussed in Section 6 for errors in chroma-gain and in γ_2) in which the input errors in toleranced quantities produce subjective effects. The particular cases for which numerical substitution will have to be made will be those for which $\Delta\xi$, $\Delta\eta$ and $\Delta Y'_2$ have numerically extreme values, and these values can be determined solely by consideration of Equations (32) and the extreme discrepancies between γ_1 and γ_2.

In Equations (33), (34) and (35), when $\Delta\xi = \Delta\eta = \Delta Y' = 0$, there will be differences between R and R_o, G and G_o and B and B_o respectively (because in general $\gamma_2 \neq \gamma_1$) and the treatment discussed above will regard these differences as contributing to ΔE_k or ΔE, although it is widely believed that an overall gamma (that is, ratio of γ_2 to γ_1) slightly greater than unity is subjectively desirable. Such considerations may mean that Equation (33) ought to be modified by the addition of a constant term on the right-hand side – possibly $(R_o{}^{1/\gamma_2} - R_o{}^{1/\gamma_1})$, and similarly for Equations (34) and (35). This possibility is not pursued here pending the formal justification of this opinion by further experimental work. The numerical handling of these equations, however, would vary very little according to the particular set of values of these constants which was ultimately preferred.

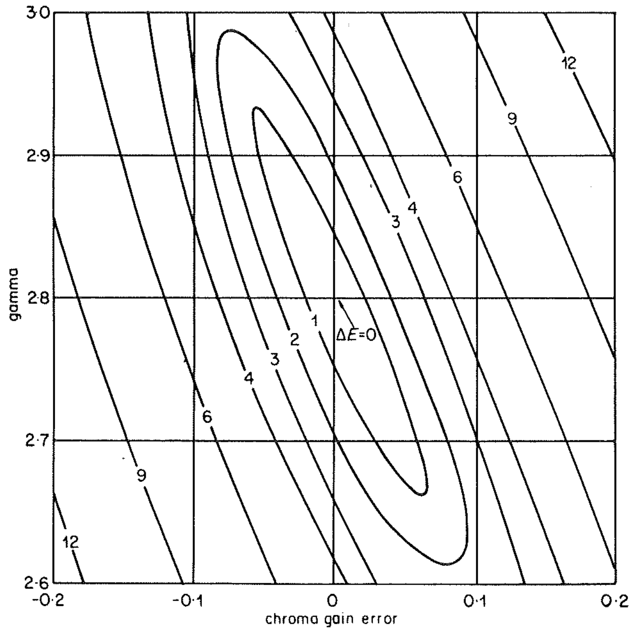

Fig. 7 - Contours of ΔE for Colour No. 9 in j.n.d. units (for varying chroma-gain error and display gamma)

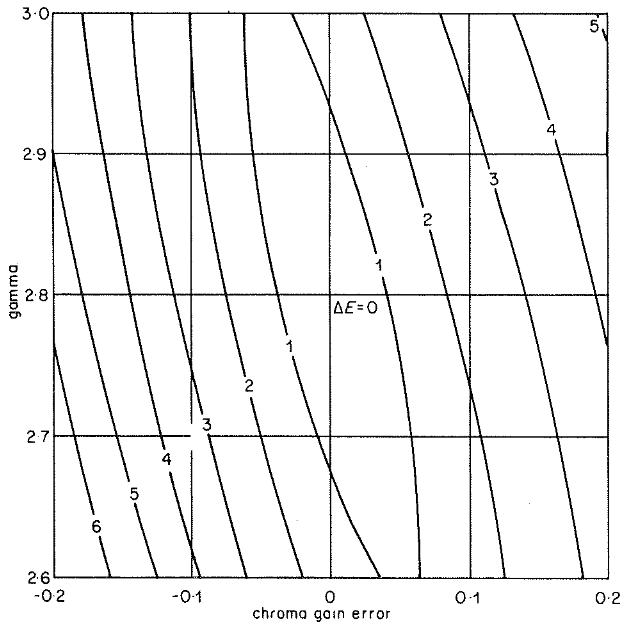

Fig. 8 - Contours of ΔE for Colour No. 12 in j.n.d. units (for varying chroma-gain error and display gamma)

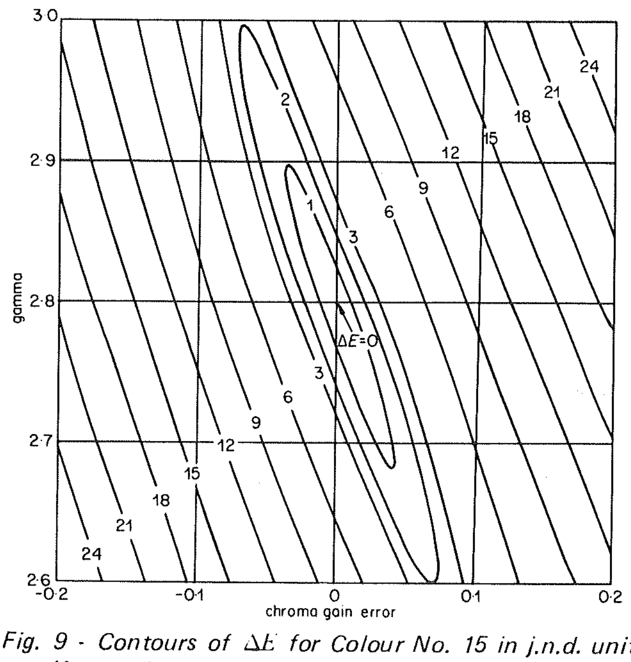

Fig. 9 - Contours of ΔE for Colour No. 15 in j.n.d. units (for varying chroma-gain error and display gamma)

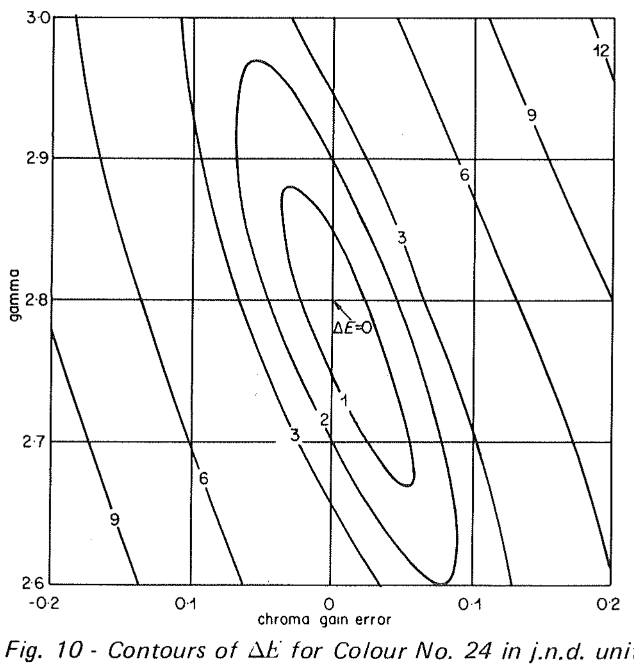

Fig. 10 - Contours of ΔE for Colour No. 24 in j.n.d. units (for varying chroma-gain error and display gamma)

8. Errors produced by various toleranced quantities

The following Table (Table 1) indicates more clearly the sort of coefficients to be expected in Equations (32) for the three principal toleranced quantities leading to large-area errors in displayed luminance and chromaticity co-ordinates. If several of the toleranced quantities listed are simultaneously in error, the overall values of $\Delta\xi$ and $\Delta\eta$ are added linearly and combined with the appropriate values of $\Delta Y'$ and $\gamma_1 - \gamma_2$ (if any). Effects of other toleranced quantities (Nos. 4 – 9 in the Table) are briefly discussed qualitatively in terms of their similarity to the effects of the principal quantities. Any errors in camera gamma or display-tube gamma are assumed to make no significant contribution to $\Delta\xi$, $\Delta\eta$ or $\Delta Y'$.

Luminance non-linearity is assumed to make only a contribution to $\Delta Y'$ related to Y'_o. Of the remaining toleranced quantities, Nos. 4, 5, 6 and 9 are assumed to affect significantly only $\Delta\xi$ and/or $\Delta\eta$, and not $\Delta Y'$, γ_1, or γ_2, while Nos. 7 and 8 are assumed to effect only $\Delta Y'$ significantly.

TABLE 1

Effect of Errors in Various Toleranced Quantities

	Toleranced Quantity	Effect on $\Delta\xi$	Effect on $\Delta\eta$
1.	Chroma-Gain Error (g or $100g\%$)	$g\xi_o$	$g\eta_o$
2.	Chrominance Signal Phase errors with respect to the mean phase of the bursts, independent of luminance signal magnitude and chrominance quadrature errors and measured near to, or at, blanking level (Angular error β radians)	$-\xi_o(1-\cos\beta)$ $\approx -\xi_o\beta^2/2$	$-\eta_o(1-\cos\beta)$ $\approx -\eta_o\beta^2/2$
3.	Error in phase quadrature between ξ and η component† of the chrominance signal (Angular Error ϵ radians)	Either $-\xi_o(1-\cos\epsilon)$ $\approx -\xi_o\epsilon^2/2$	0
		or 0	$-\eta_o(1-\cos\epsilon)$ $\approx -\eta\epsilon^2/2$

Other relevant toleranced quantities are:

4. Error in the ratio between the amplitude of the colour burst and that of the chrominance signal (Broadly similar to chroma-gain, No. 1 above).

5. Differential Gain
(Related to chroma-gain error, No. 1 above, but g is a function of Y'_o)

6. Differential Phase
(Related to chrominance signal phase error, No. 2 above, but β is a function of Y'_o).

7. The so-called 'line-time non-linearity'
(There is a non-linear relation between Y' and Y'_o, but chrominance is not affected. Broadly similar to luminance non-linearity.)

8. Chrominance-to-luminance crosstalk
(A distortion of luminance which depends upon $(\xi_o{}^2 + \eta_o{}^2)$. The effect is broadly similar to that of luminance non-linearity.)

9. Chrominance-to-luminance gain inequality. (Broadly similar to chroma-gain, No. 1 above.)

Detailed analysis of items 4 to 9 is beyond the scope of this report, but could be undertaken by similar methods.

† Usually called U and V components, but see list of symbols.

9. Discussion and conclusions

The results derived above indicate the general way in which two particular toleranced quantities simultaneously in error translate these errors into subjectively appreciable effects on ΔE_k or ΔE. The general case of several toleranced quantities in error simultaneously has been discussed in outline: the key equations are (32) which relate the causing errors to $\Delta\xi$, $\Delta\eta$, $\Delta Y'$ and (33), (34) and (35) which give the changes in R^{1/γ_2}, G^{1/γ_2} and B^{1/γ_2} in terms of the changes in $\Delta\xi$ etc. Errors in γ_1 and γ_2, if any, must also be allowed for.

From Equations (32) it will usually be a straightforward operation to determine the extreme numerical values of $\Delta\xi$, $\Delta\eta$ etc. which must be taken into account. (Initially the assumption that a toleranced quantity is doing its worst when its error is numerically equal to 90% of the maximum allowed is probably adequate.) Hence in practice there will only be a small number of numerical cases for which it is ever necessary actually to calculate ΔE_k or ΔE at all; even then, the behaviour of the predominant component of ΔE_k may provide all the significant information. Should a large-scale investigation be necessary there should be little difficulty in constructing a computer programme for determining either ΔE_k or ΔE. At present it is not at all obvious which of these quantities is the best objective measure of subjective reaction to resultant errors in toleranced quantities, although results so far obtained indicate that ΔE is superior to ΔE_k. The main conclusion arising from this investigation is that it does not seem to be necessary to consider the statistical distribution of toleranced quantities within their tolerances or methods of combining such distributions when several toleranced quantities are simultaneously in error. If a relevant value of ΔE_k (or ΔE) is n times what it ought to be, the obvious corrective action to try first is to reduce in the ratio n : 1 the tolerances of the quantities principally responsible.

10. Acknowledgements

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